Spring semester 2012
January 19, 2012

# MATHEMATICS DEPARTMENT CONTEST

The Department of Mathematics invites all CUA students to compete, for the fun of it, in a mathematics contest. The contest consists of mathematical problems or puzzles which can be understood by anyone with the usual high school mathematics background. The most successful contestants will be invited to the Mathematics Department end-of-semester party to receive prizes. There will be prizes for the students who solve the most problems and for those who submit the most interesting or original solutions (even if for only one problem). Send your solutions by **April 18, 2012** to Dr. Alexander Levin at the Mathematics Department in room 207 MCM. They need not be typed but should be legible and should show or explain how you solved the puzzle.

**Problem 1.** There are 101 soldiers in a division. It is known that there was no combat operation where all 101 soldiers participated, and that every two soldiers participated in the same combat operation exactly once. Prove that there is a soldier who participated in more than ten combat operations.

**Problem 2.** Three elder people, John, Frank, and Peter live in different houses. They can walk at the speed of 1, 2, and 3 miles per hour, respectively. They want to choose a place for their daily meeting in such a way that the sum of times each of them would spend to reach this place from his home will be minimum possible. Where should they arrange their meetings?

**Problem 3.** A student did not noticed the sign of multiplication between two seven-digit numbers and wrote a single fourteen-digit number. It turned out that this fourteen-digit number was three times greater than the product of the two original seven-digit numbers. Find these seven-digit numbers.

**Problem 4.** Find all irreducible fractions $\frac{a}{b}$ ($a$ and $b$ are positive integers) that can be also written as $b.a$ (the decimal point separates the decimal representations of $b$ and $a$).

**Problem 5.** Let us consider an isosceles triangle $ABC$ where $AB = BC$. Suppose that $M$ is a point on the side $BC$ such that the segment $AM$ divides the triangle in two isosceles triangles with the bases $AB$ and $MC$ ($AM = MB$ and $AC = AM$). Find the angle $\angle ABC$.

**Problem 6.** There are three secret positive integers $a, b$, and $c$. A mathematician $X$ has found out the sum $S$ of the numbers, while a mathematician $Y$ has found out the product $P$ of these numbers. $X$ said, "If I knew that your number $P$ is greater than my number $S$, I would determine the secret numbers." "Actually, my number is less than yours, but now I know $a, b$, and $c$," $Y$ replied. What are the secret numbers?