

Fall semester 2014  
August 25, 2014

## MATHEMATICS DEPARTMENT CONTEST

The Department of Mathematics invites all CUA students to compete, for the fun of it, in a mathematics contest. The contest consists of mathematical problems or puzzles which can be understood by anyone with the usual high school mathematics background. The most successful contestants will be invited to the Mathematics Department end-of-semester party to receive prizes. There will be prizes for the students who solve the most problems and for those who submit the most interesting or original solutions (even if for only one problem).

Send your solutions by **November 18, 2014** to Dr. Alexander Levin at the Mathematics Department in McMahan Hall, room 207. They need not be typed but should be legible and should show or explain how you solved the puzzle.

**Problem 1.** Prove that if one chooses 101 numbers from the first two hundred natural numbers 1, 2, 3, ..., 200, then the sample will contain two numbers one of which divides the other one.

**Problem 2.** There are  $mn$  positive integers arranged in a rectangular table with  $m$  rows and  $n$  columns. It is allowed to subtract 1 from each number of the same row and to multiply all numbers of the same column by 2. Prove that using these operations one can obtain a table of all zeros.

**Problem 3.** There are 189 stones arranged in three piles containing 21, 81, and 87 stones, respectively. It is allowed to merge two piles together and to divide a pile with an even number of pebbles into two equal piles. Is there a sequence of such operations that would result in 189 piles with one pebble each?

**Problem 4.** Does there exist a set of 2014 positive integers such that the sum of any 2013 of these numbers is divisible by the remaining number? Justify your answer.

**Problem 5.** Let us consider a triangle whose sides has lengths  $a$ ,  $b$  and  $c$ . Prove that

$$a^3 + b^3 + 3abc > c^3.$$

**Problem 6.** Two equal circles of radius less than 1 in with centers  $O_1$  and  $O_2$  touch each other, and each of them touches a third circle of radius 5 in from inside. If  $O$  is the center of the third circle, what is the perimeter of the triangle  $O_1O_2O$ ?