Spring semester 2014
January 31, 2014

MATHEMATICS DEPARTMENT CONTEST

The Department of Mathematics invites all CUA students to compete, for the fun of it, in a mathematics contest. The contest consists of mathematical problems or puzzles which can be understood by anyone with the usual high school mathematics background. The most successful contestants will be invited to the Mathematics Department end-of-semester party to receive prizes. There will be prizes for the students who solve the most problems and for those who submit the most interesting or original solutions (even if for only one problem).

Send your solutions by **April 14, 2014** to Dr. Alexander Levin at the Mathematics Department in McMahon Hall, room 207. They need not be typed but should be legible and should show or explain how you solved the puzzle.

Problem 1. Seven teams participated in a soccer tournament where every team plays one game with every other team. According to the rules, the winner of a game earns 3 points, the loser gets no points, and in the case of draw, each team get 1 point. At the end of the tournament, the teams had 14, 13, 9, 8, 7, 4, and 3 points. How many games ended in a draw?

Problem 2. It is known that the product of two positive real numbers is greater than their sum. Prove that the sum of the numbers is greater than 4.

Problem 3. Is it possible to construct a convex polygon such that the number of its diagonals is ten times the number of its sides? Justify your answer. (A polygon is said to be convex if for any two points A and B inside the polygon, the whole segment AB lies inside this polygon. For example, all triangles, parallelograms and trapezoids are convex polygons. As usual, by a diagonal we mean a segment connecting two vertices of a polygon that do not belong to the same side.)

Problem 4. Is there a real number a such that the numbers $a + \sqrt{15}$ and $\sqrt{15} - \dfrac{1}{a}$ are both integers? Justify your answer.

Problem 5. There are 25 students in some class. It is known that in any group of three students, there are two friends. Prove that there is a student who has more than 11 friends in the class.

Problem 6. There are 2014 points and a circle of radius 1 in in the plane. Prove that there is a point A on the circumference of the circle such that the sum of 2014 distances from A to the points is greater than or equal to 2014 in.