The Department of Mathematics invites all CUA students to compete, for the fun of it, in a mathematics contest. The contest consists of mathematical problems or puzzles which can be understood by anyone with the usual high school mathematics background. The most successful contestants will be invited to the Mathematics Department end-of-semester party to receive prizes. There will be prizes for the students who solve the most problems and for those who submit the most interesting or original solutions (even if for only one problem).

Send your solutions by **April 25, 2016** to Dr. Alexander Levin at the Mathematics Department in Aquinas Hall, room 116. They need not be typed but should be legible and should show or explain how you solved the puzzle.

**Problem 1.** Twelve hockey teams have participated in a tournament where every team played with any other team exactly once. It is known that two teams won seven games each and there was no overtime in any game. Prove that there are three teams, A, B, and C, such that A has beaten B, B has beaten C, and C has beaten A.

**Problem 2.** The set of nine numbers \(\{1, 2, 3, 4, 5, 6, 7, 8, 9\}\) has been divided into three groups (two different groups have no common numbers). Prove that there is a group such that the product of all its numbers is greater than or equal to 72.

**Problem 3.** Is it possible to cover an equilateral triangle with two other equilateral triangles whose sides are shorter than the sides of the original triangle? Justify your answer.

**Problem 4.** Find all prime numbers \(p\) such that \(2p^4 - p^2 + 16\) is a square of some integer.

**Problem 5.** Find all positive integers \(x, y,\) and \(z\) such that the equality \(x^n + y^n = z^{n+1}\) holds for any positive integer \(n\).

**Problem 6.** Prove that if \(a\) and \(b\) are the lengths of the legs of a right triangle and \(c\) is the length of its hypothenuse, then \(ab(a + b + c) < \frac{5}{4} c^3\).