# MATHEMATICS DEPARTMENT CONTEST

## Fall semester 2017

The Department of Mathematics invites all CUA students to compete, for the fun of it, in a mathematics contest. The contest consists of mathematical problems or puzzles which can be understood by anyone with the usual high school mathematics background. The most successful contestants will be invited to the Mathematics Department end-of-semester party to receive prizes. There will be prizes for the students who solve the most problems and for those who submit the most interesting or original solutions (even if for only one problem).

Send your solutions by **November 27, 2017** to Dr. Alexander Levin at the Mathematics Department in Aquinas Hall, room 116. They need not be typed but should be legible and should show or explain how you solved the puzzle.

**Problem 1.** Prove that there is a positive integer $n$ such that the number $3^n$ ends with 00001.

**Problem 2.** Positive integers $1, 2, \ldots, 10$ are written in a row. Is it possible to place signs $+$ or $-$ between every two consecutive numbers in order to get 0? Justify your answer.

**Problem 3.** John has a balance scale that shows the difference (in grams) between the weights on its two cups. He has received 101 coins and he knows that 50 of them are counterfeit. He also knows that a counterfeit coin is either one gram lighter or one gram heavier than a genuine one. If John chooses a coin at random, can he determine whether the chosen coin is genuine in just one weighing? Justify your answer.

**Problem 4.** Let a point $D$ lie in the middle of the base $BC$ of an isosceles triangle $ABC$. Let $M$ be a point on the side $AB$ such that the distance from $M$ to $A$ is two times shorter than the distance from $M$ to $B$. It is also known that the length of the segment $AD$ is equal to the length of the side $BC$. Find the angle $\angle MCB$.

**Problem 5.** If 30 students from the same class visit a movie theater and take seats arbitrarily, then there will be at least two classmates in every row. If there are 26 students in a class and they take seats arbitrarily in the same movie theater, then there will be always at least three rows with no students from this class. How many rows are in the movie theater? Justify your answer.

**Problem 6.** Let $a$, $b$, $c$, and $d$ be four positive real numbers. Prove that

$$(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right) \geq 16.$$